

## BOOK REVIEW

**Inverse Eigenvalue Problems: Theory, Algorithms, and Applications.** By MOODY T. CHU & GENE H. GOLUB. Oxford University Press, 2005. 387 pp. ISBN 0-19-856664-6. £60.00

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A matrix inverse eigenvalue problem has the form: ‘given a set of eigenvalues, find a matrix having these eigenvalues’. The number of specified eigenvalues may be equal to the dimension of the matrix or less than it (allowing freedom in the unspecified eigenvalues). The matrix is usually required to have a specific structure – for example, some combination of real, tridiagonal, non-negative, symmetric, and Toeplitz. Such problems arise in a wide variety of settings where it is required to reconstruct the parameters of a system from knowledge of its behaviour.

The subject of matrix inverse eigenvalue problems has grown over the years and now encompasses a broad range of problems. In addition to inverse eigenvalue problems *per se*, Chu & Golub treat inverse singular value problems, problems with least squares constraints on the spectrum or the matrix structure, and rank constrained matrix approximation problems.

This research monograph presents a timely survey of the subject, treating the mathematical questions of existence and uniqueness, algorithms for solving the various problems, and sensitivity issues. Chapter 2 describes a long list of applications in areas such as control theory, mechanics, signal processing and numerical analysis. Subsequent chapters cover ‘parametrized’, ‘structured’, ‘partially described’ and ‘least squares’ inverse eigenvalue problems, the last three chapters being titled ‘Spectrally constrained approximation’, ‘Structured low rank approximation’ and ‘Group orbitally constrained approximation’. As well as single matrix eigenvalue problems  $Ax = \lambda x$ , inverse quadratic eigenvalue problems  $(\lambda^2 A + \lambda B + C)x = 0$  are treated in some detail in §5.3. The book has no exercises but does contain numerical examples.

A couple of points are worth making about the usability of the book. At the start of Chapter 2 the authors say “Because of the vast diversity of problems, theories, algorithms, and open questions, it is very difficult to maintain any uniformity in the presentation”. Indeed I found that leafing through the book I struggled to keep track of my place in the overall structure, not helped by the omission of chapter and section numbers from the page headers. In an attempt to summarize the nomenclature for the wide variety of problems considered, the authors define a list of 34 acronyms (AIEP, ECIEP, ISEP, . . . , UHIEP), with a summary on page xiv, and use these acronyms throughout. I remain unconvinced about the use of this long and rather un-memorable list, and suspect it will take the reader some time to become comfortable with it.

An extensive bibliography is included, but, rather oddly, the authors say on page 9 that “the list is far from being comprehensive as we have already overlooked much of the engineering literature”. The book is generally well written, though with occasionally stilted language, as in this sentence from the preface: “Though some of these chapters imbricate and refer to each other, readers might find some relief in knowing that each chapter can be rendered independently of each other”.

Overall, Chu & Golub have provided a thorough treatment of the growing subject of inverse eigenvalue problems. The book collects together much material not previously available in book form and benefits from both authors' many years of experience of working on inverse eigenvalue and related problems. I recommend the book to anyone who wishes to know more about the subject.

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